

Traveling Salesman

Given a set of cities $\{1, \dots, n\}$ and a symmetric matrix $C = (c_{ij})$, $c_{ij} \geq 0$ that specifies for every pair $(i, j) \in [n] \times [n]$ the cost for travelling from city i to city j . Find a permutation π of the cities such that the round-trip cost

$$c_{\pi(1)\pi(n)} + \sum_{i=1}^{n-1} c_{\pi(i)\pi(i+1)}$$

is minimized.

Traveling Salesman

Theorem 3

There does not exist an $O(2^n)$ -approximation algorithm for TSP.

Hamiltonian Cycle:

For a given undirected graph $G = (V, E)$ decide whether there exists a simple cycle that contains all nodes in G .

Given an instance to HAMILTONIAN, we create an instance for TSP.

If $|V| = 1$, then set c_{ij} to 0. Otherwise, set c_{ij} to 1. This way, the instance has polynomial size.

There exists a Hamiltonian Cycle if and only if there exists a tour with cost n . Our tiny tour has cost strictly larger than n .

No $O(2^n)$ -approximation algorithm could decide our case. The smallest tour cannot exist unless $P = NP$.

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- ▶ Given an instance to HAMPATH we create an instance for TSP.
- ▶ If $(i, j) \notin E$ then set c_{ij} to $n2^n$ otw. set c_{ij} to 1. This instance has polynomial size.
- ▶ There exists a Hamiltonian Path iff there exists a tour with cost n . Otw. any tour has cost strictly larger than $n2^n$.
- ▶ An $O(2^n)$ -approximation algorithm could decide btw. these cases. Hence, cannot exist unless $P = NP$.

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Metric Traveling Salesman

In the metric version we assume for every triple $i, j, k \in \{1, \dots, n\}$

$$c_{ik} \leq c_{ij} + c_{jk} .$$

It is convenient to view the input as a complete undirected graph $G = (V, E)$, where c_{ij} for an edge (i, j) defines the distance between nodes i and j .

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TSP: Lower Bound I

Lemma 4

The cost $\text{OPT}_{\text{TSP}}(G)$ of an optimum traveling salesman tour is at least as large as the weight $\text{OPT}_{\text{MST}}(G)$ of a minimum spanning tree in G .

Proof:

- 1. Let T be the minimum spanning tree of G .
- 2. Let OPT_{TSP} be an optimum traveling salesman tour.
- 3. Let E be the set of edges of T .
- 4. Let E' be the set of edges of OPT_{TSP} .
- 5. Let E'' be the set of edges of G that are not in E .
- 6. Let E''' be the set of edges of G that are not in E' .
- 7. Let E''' be the set of edges of G that are not in E'' .

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TSP: Greedy Algorithm

- ▶ Start with a tour on a subset S containing a single node.
- ▶ Take the node v closest to S . Add it S and expand the existing tour on S to include v .
- ▶ Repeat until all nodes have been processed.

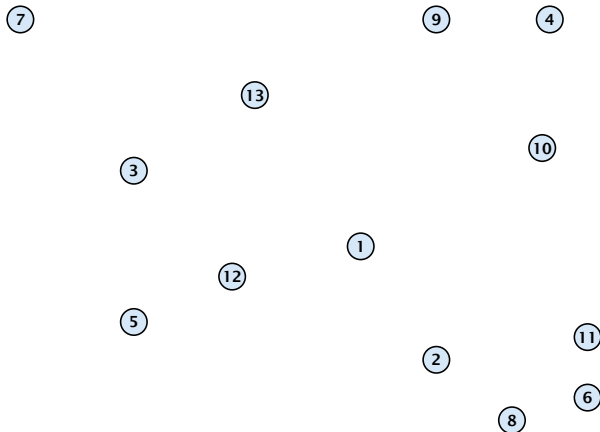
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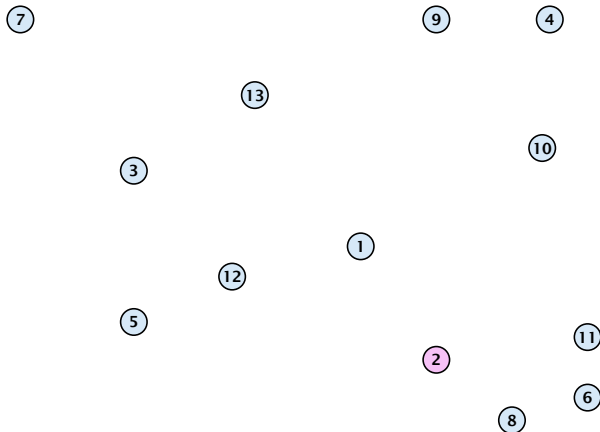
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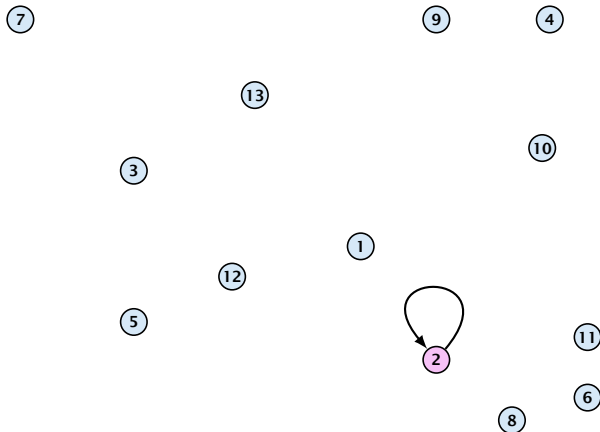
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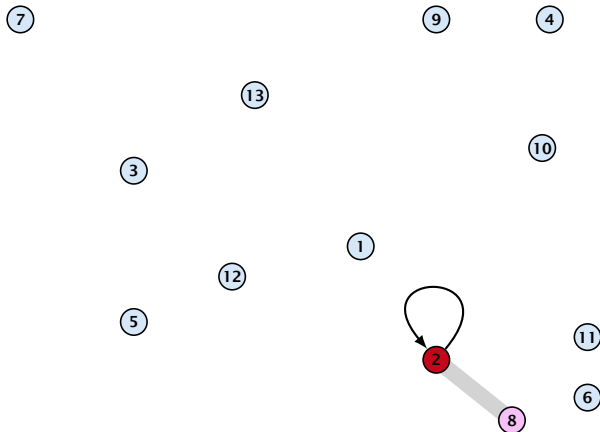
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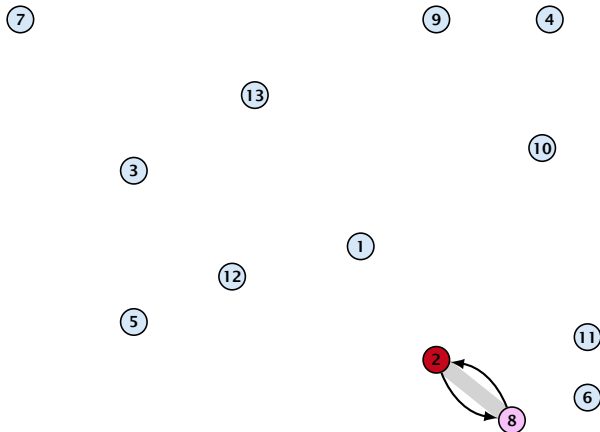
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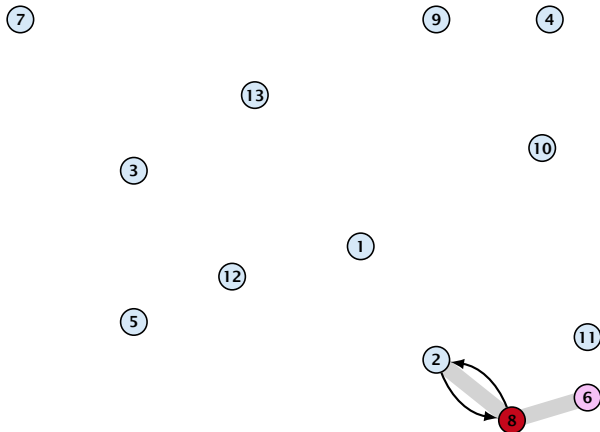
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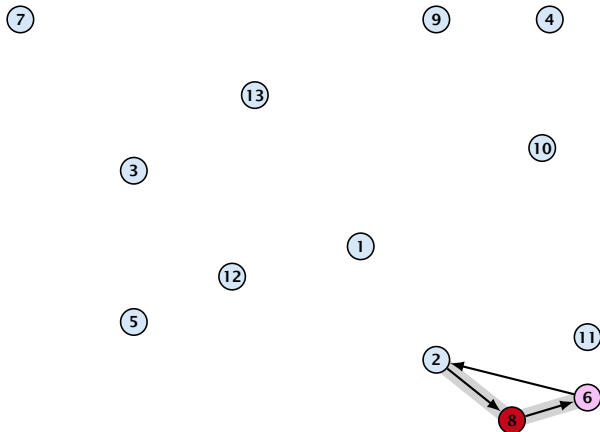
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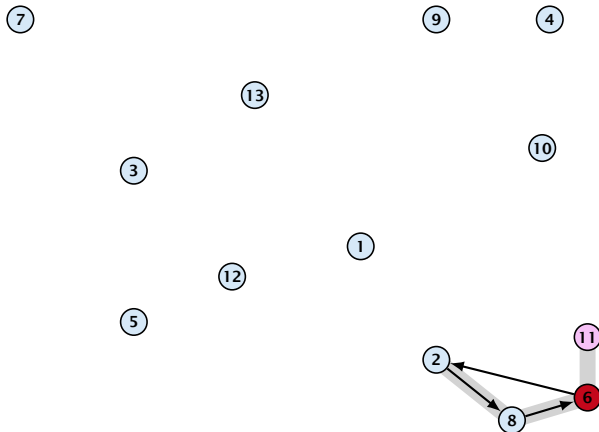
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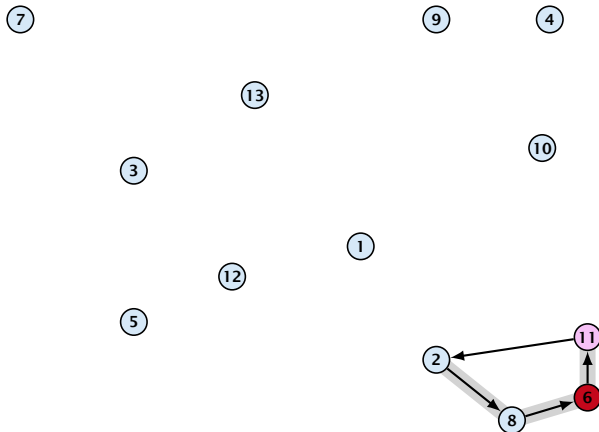
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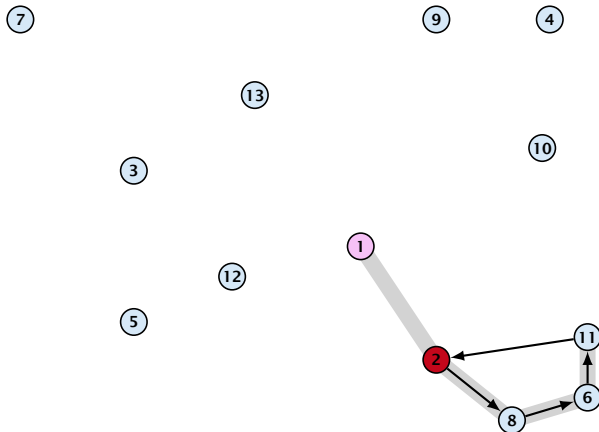
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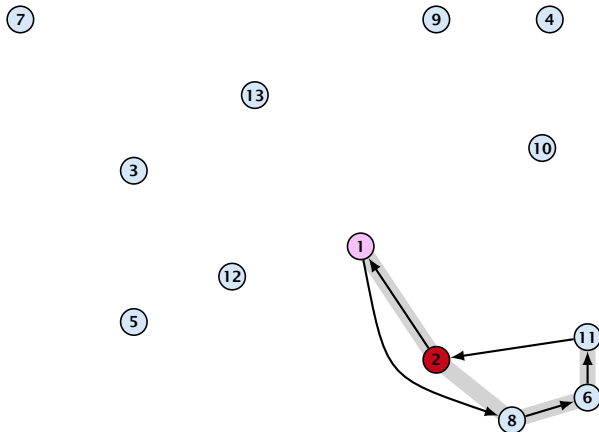
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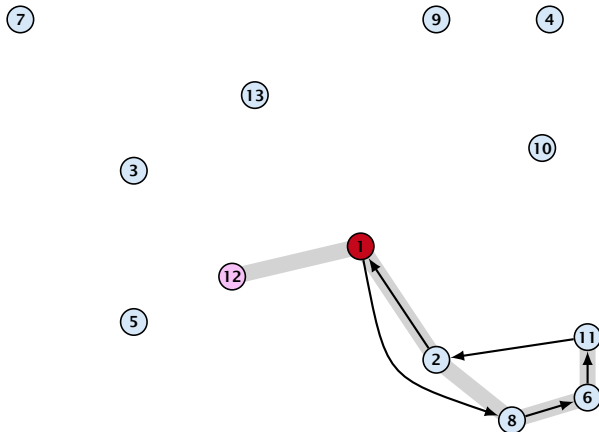
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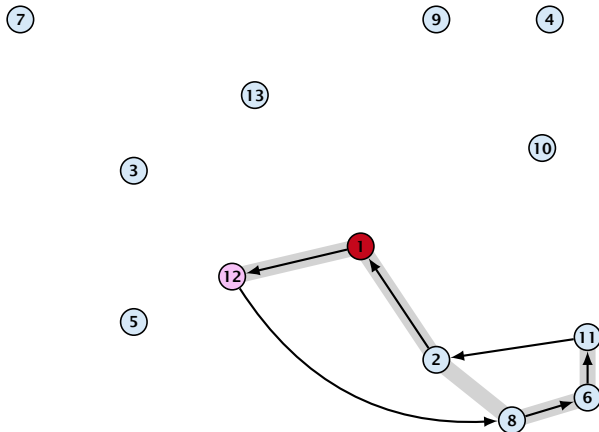
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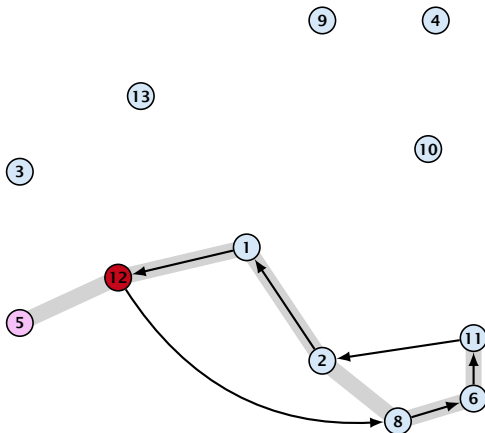
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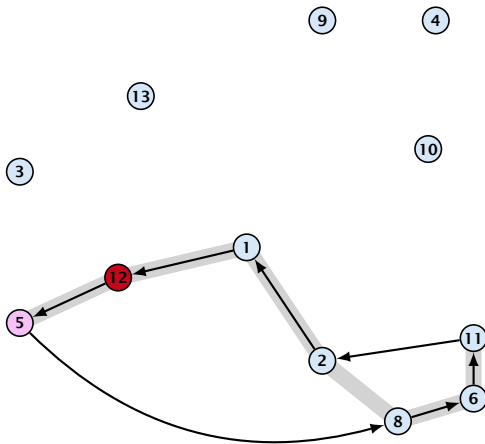
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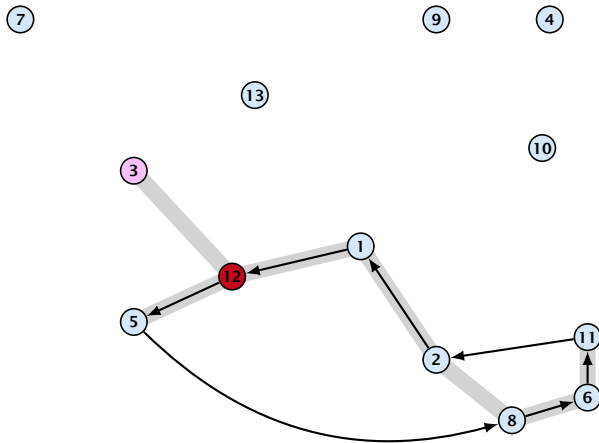
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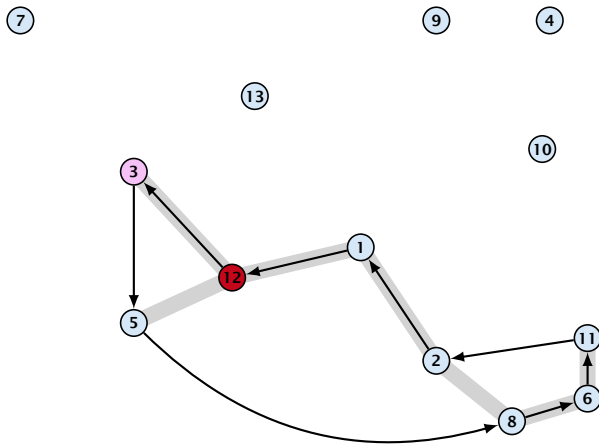
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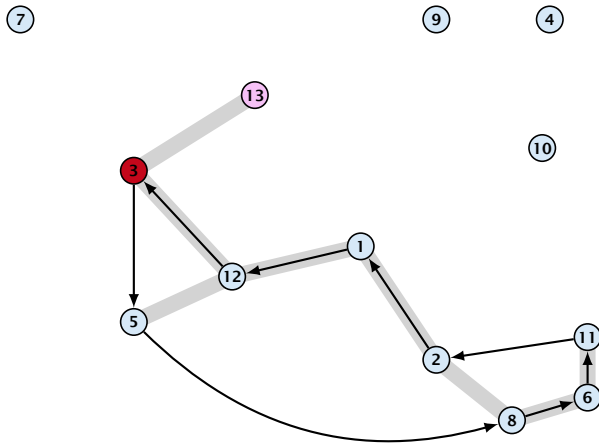
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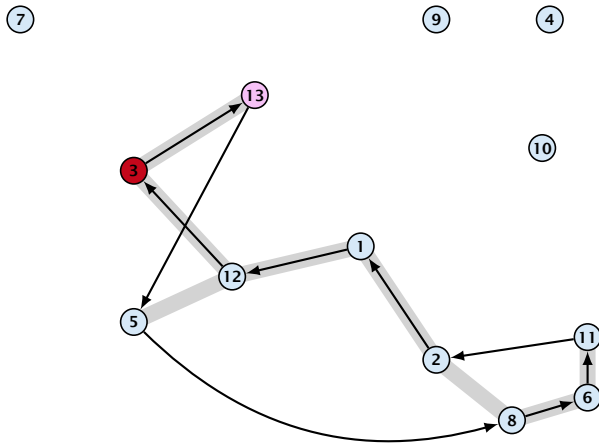
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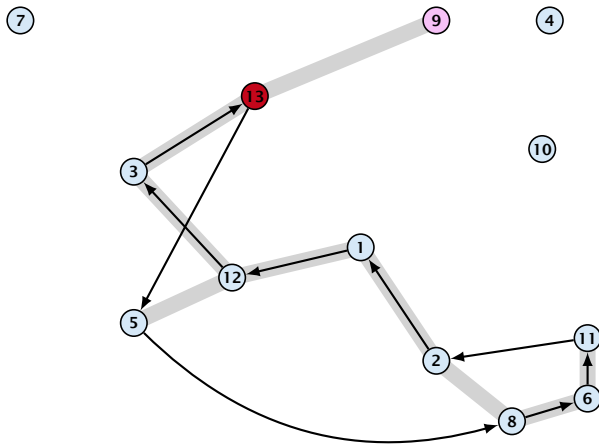
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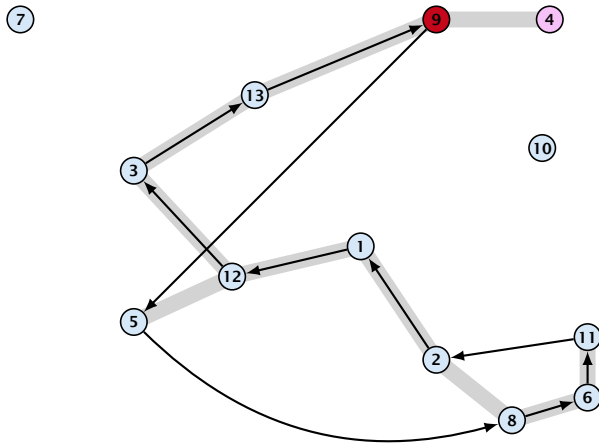


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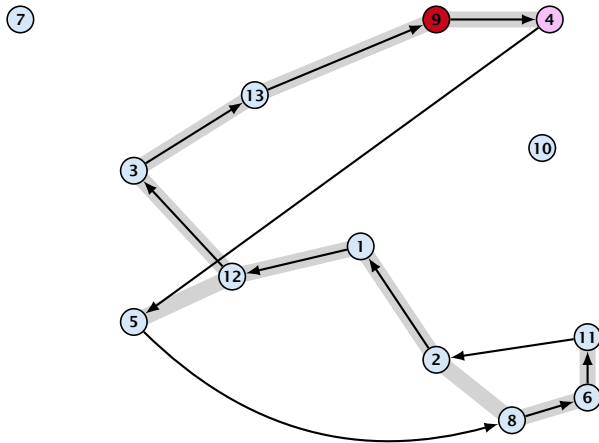
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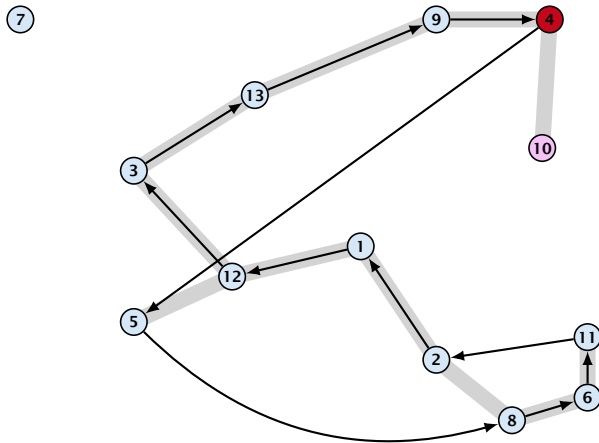
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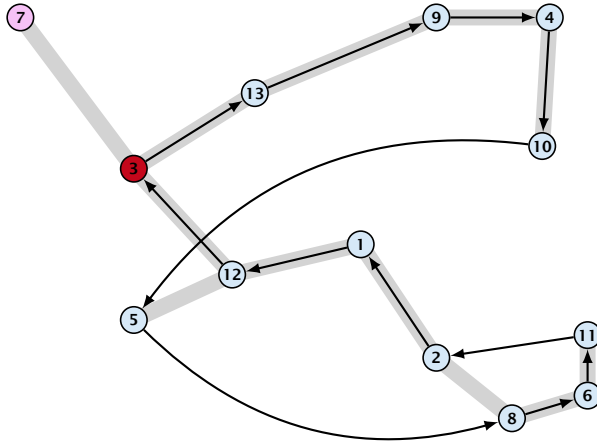


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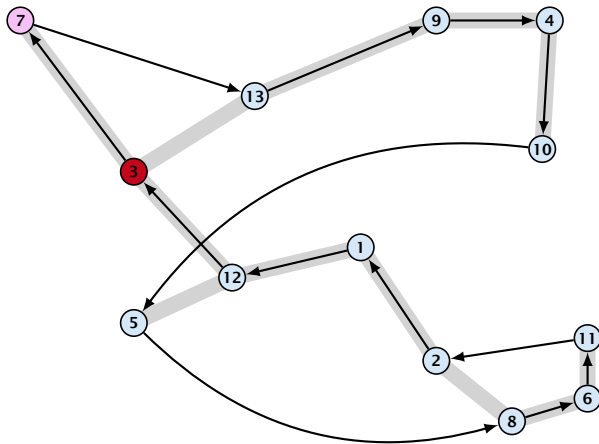
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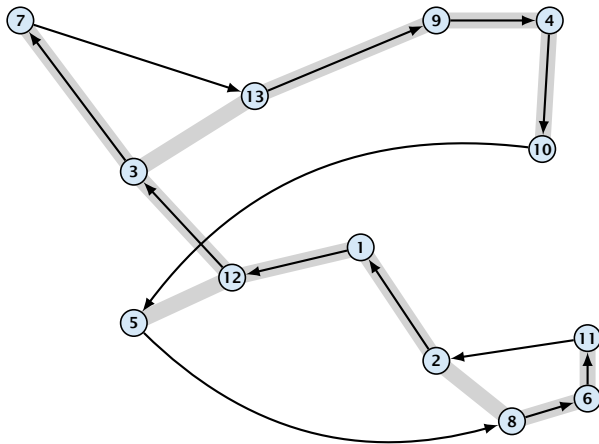
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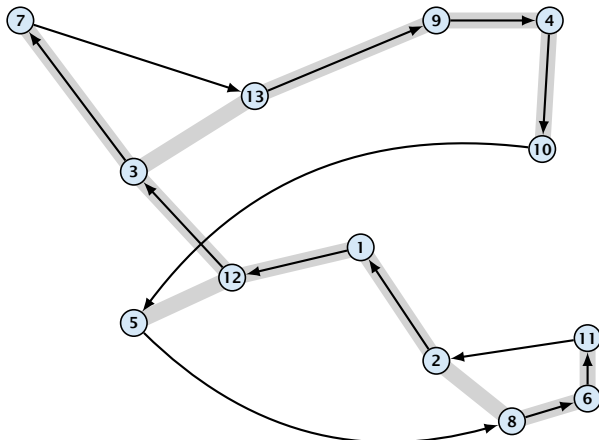
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Lemma 5

The Greedy algorithm is a 2-approximation algorithm.

Let S_i be the set at the start of the i -th iteration, and let v_i denote the node added during the iteration.

Further let $s_i \in S_i$ be the node closest to $v_i \in S_i$.

Let r_i denote the successor of s_i in the tour before inserting v_i .

We replace the edge (s_i, r_i) in the tour by the two edges (s_i, v_i) and (v_i, r_i) .

This increases the cost by

$$c_{s_i, v_i} + c_{v_i, r_i} - c_{s_i, r_i} \leq 2c_{s_i, v_i}$$

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The edges (s_i, v_i) considered during the Greedy algorithm are exactly the edges considered during PRIMs MST algorithm.

Hence,

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TSP: A different approach

Suppose that we are given an **Eulerian** graph $G' = (V, E', c')$ of $G = (V, E, c)$ such that for any edge $(i, j) \in E'$ $c'(i, j) \geq c(i, j)$.

Then we can find a TSP-tour of cost at most

$$\sum_{e \in E'} c'(e)$$

1. Find an Euler tour of G' .
2. Fix a permutation of the cities (i.e., a TSP-tour) by traversing the Euler tour and only note the first occurrence of a city.
3. The cost of this TSP tour is at most the cost of the Euler tour because of triangle inequality.

This technique is known as **short cutting** the Euler tour.

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Find an Eulerian graph G' of G .

Then the cost of the TSP-tour is at most the cost of G' , because the TSP-tour contains every edge of G' at least once.

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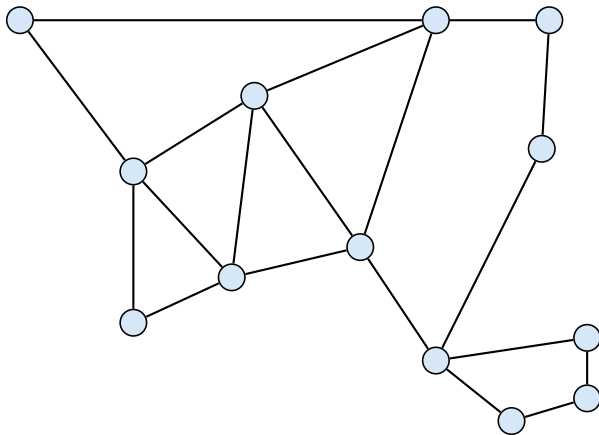
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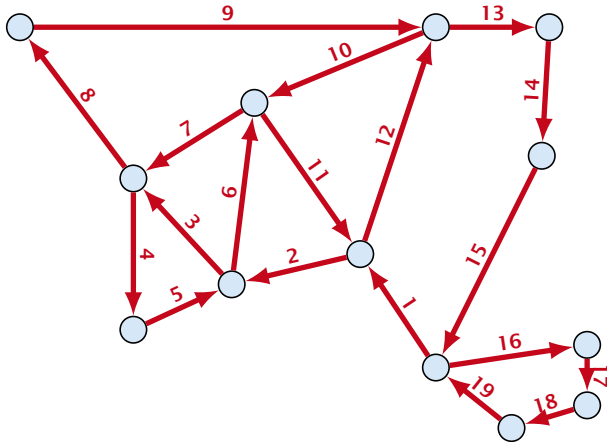
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This technique is known as **short cutting** the Euler tour.

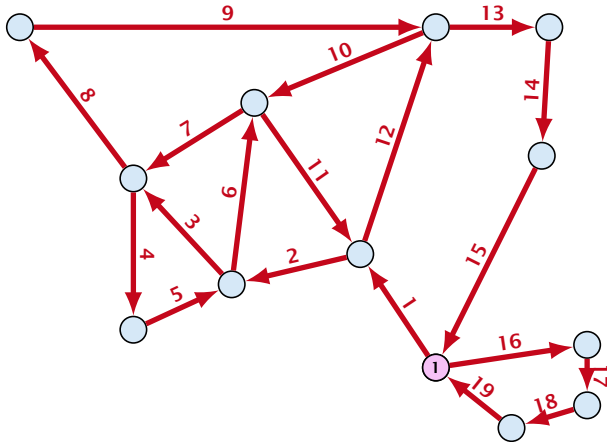
TSP: A different approach



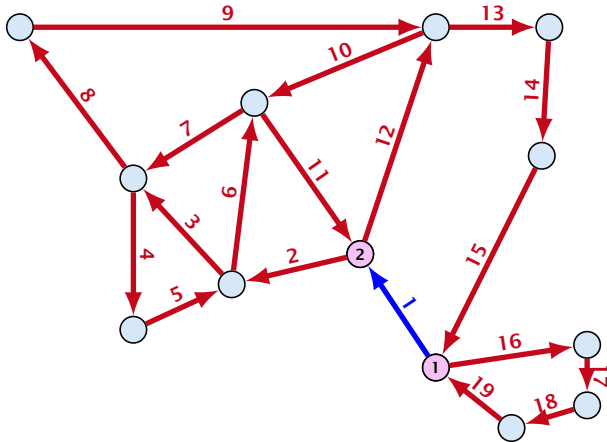
TSP: A different approach



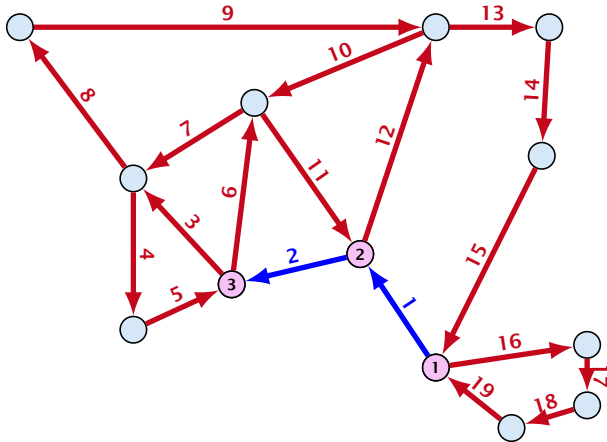
TSP: A different approach



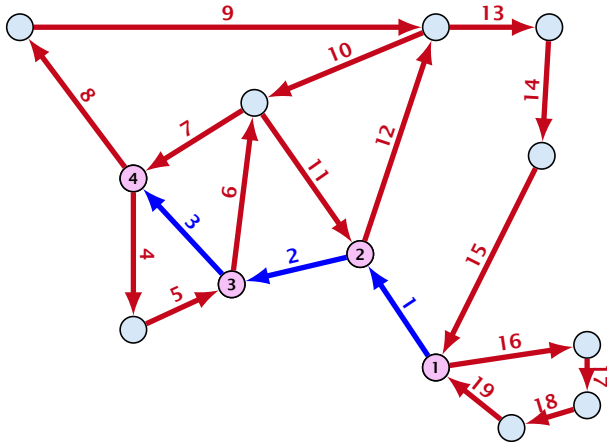
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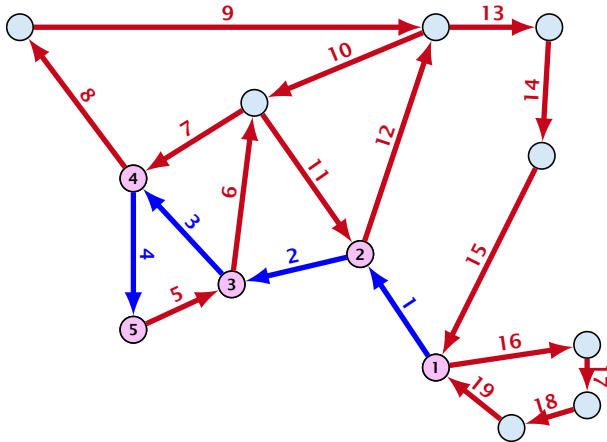
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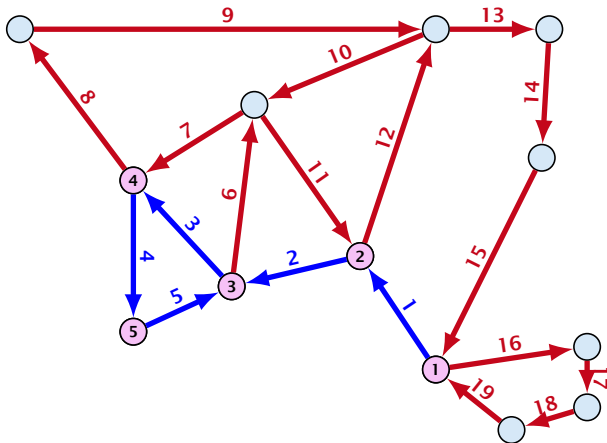
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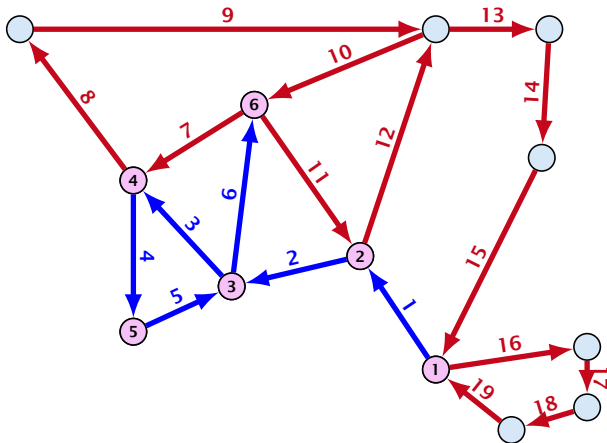
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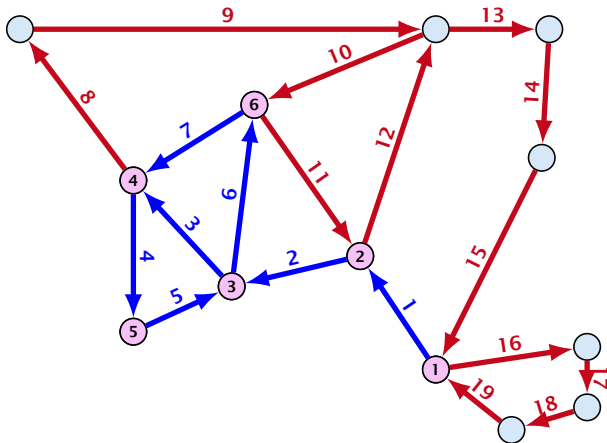
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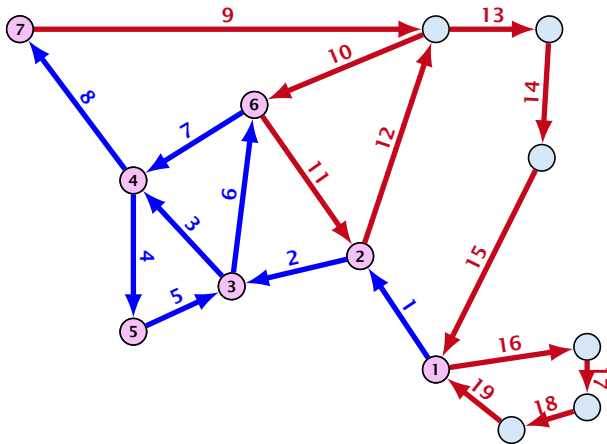
TSP: A different approach



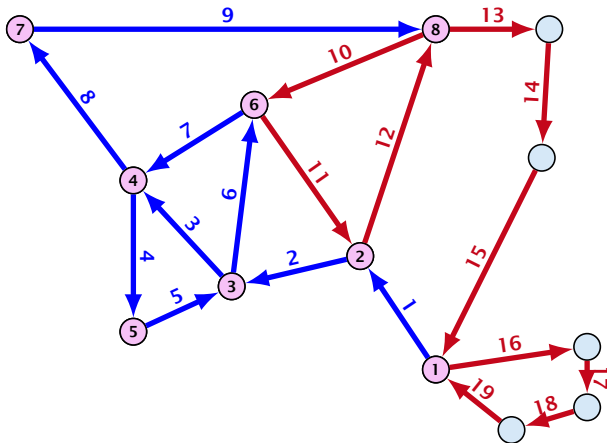
TSP: A different approach



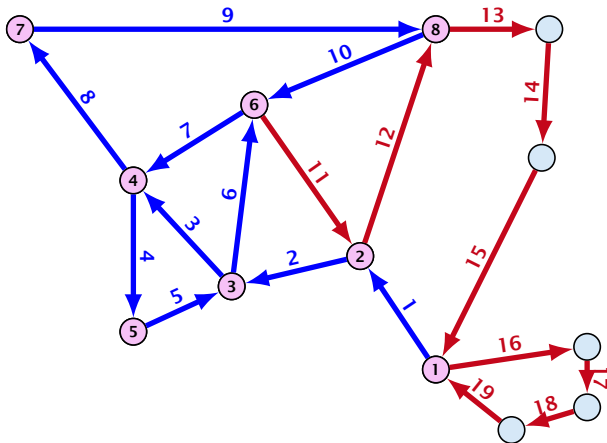
TSP: A different approach



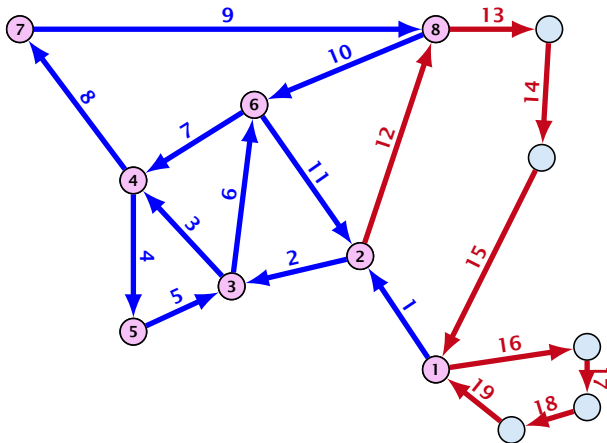
TSP: A different approach



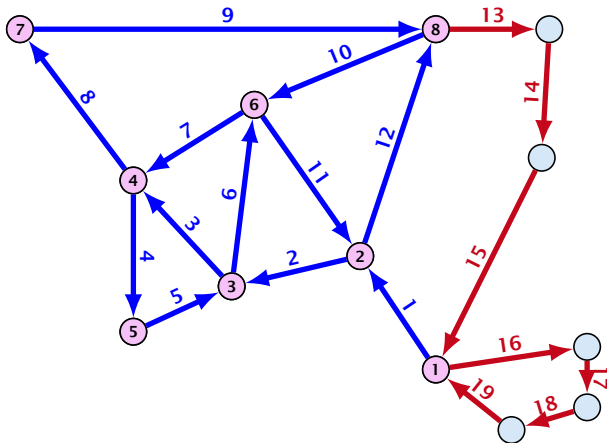
TSP: A different approach



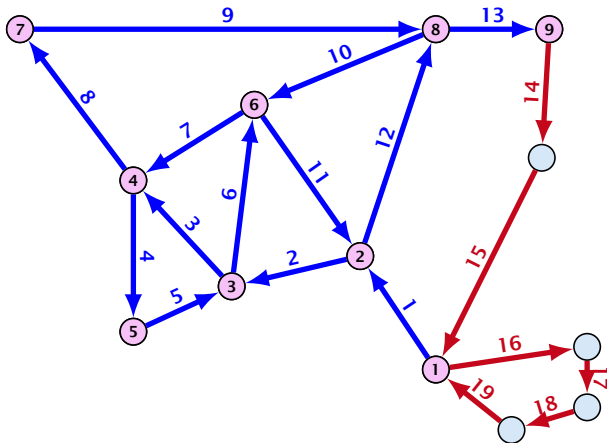
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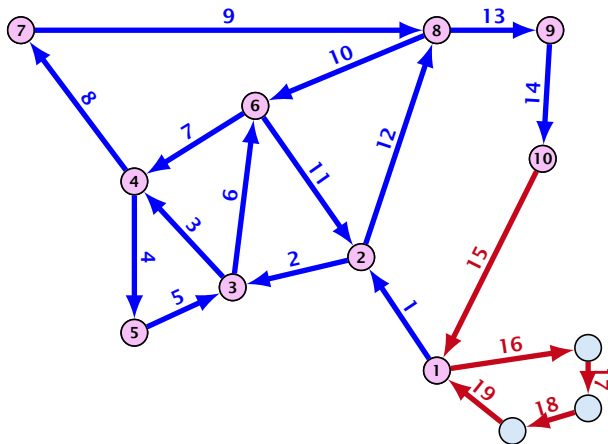
TSP: A different approach



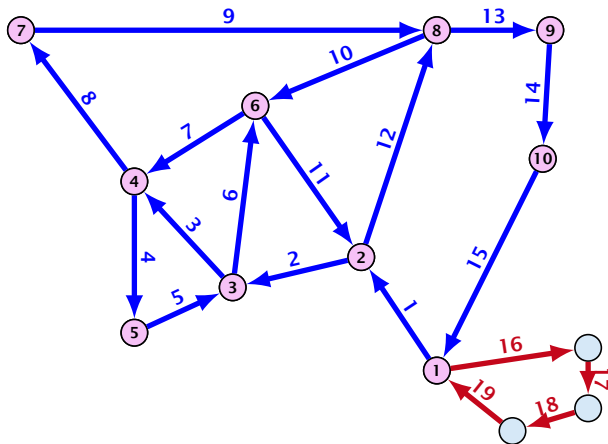
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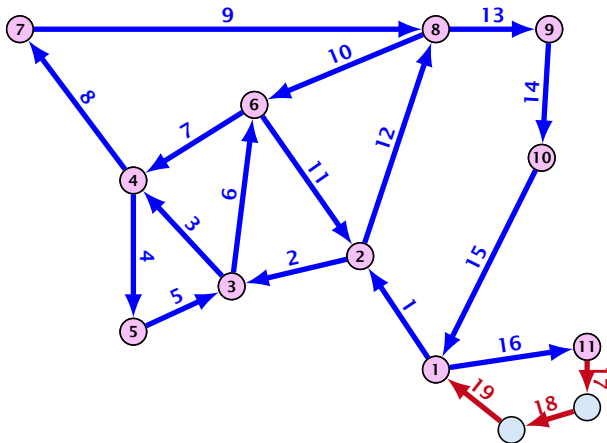
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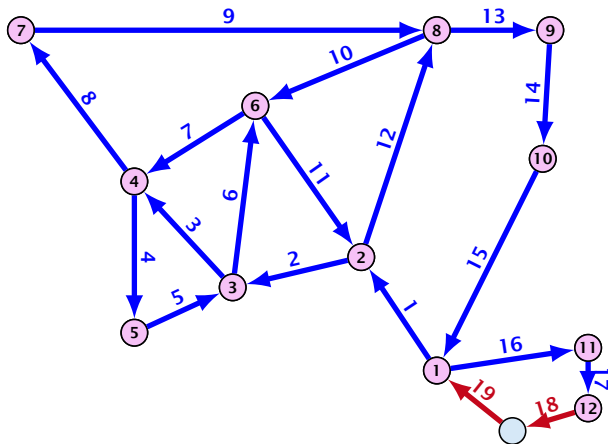
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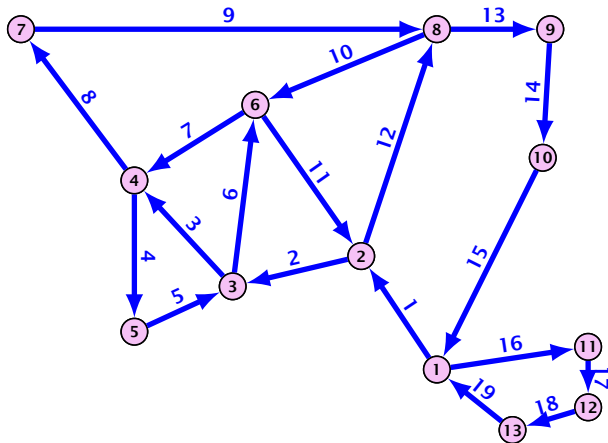
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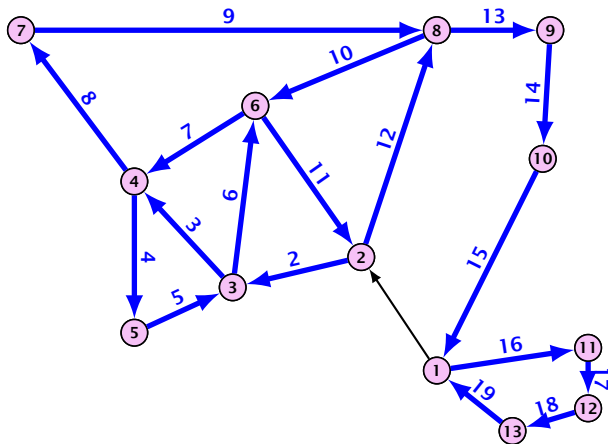
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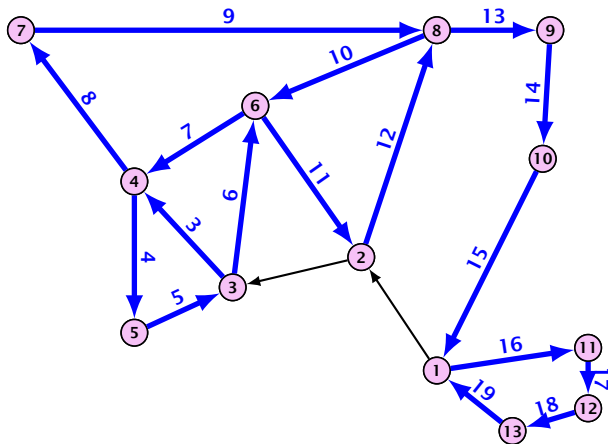
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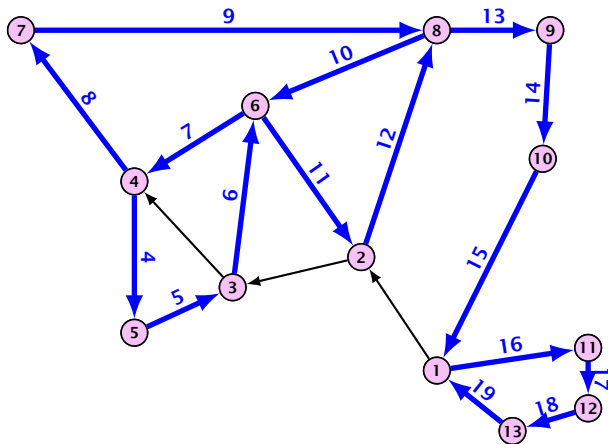
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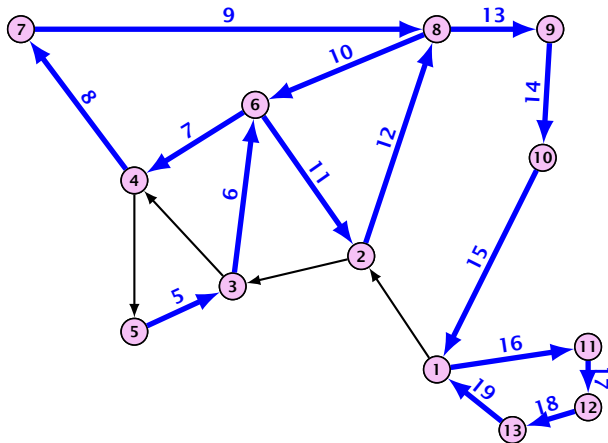
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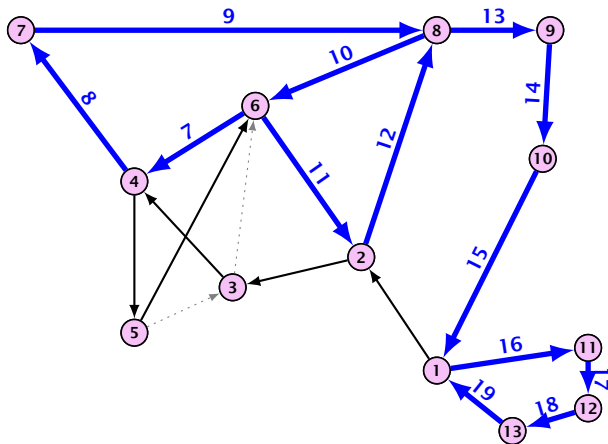
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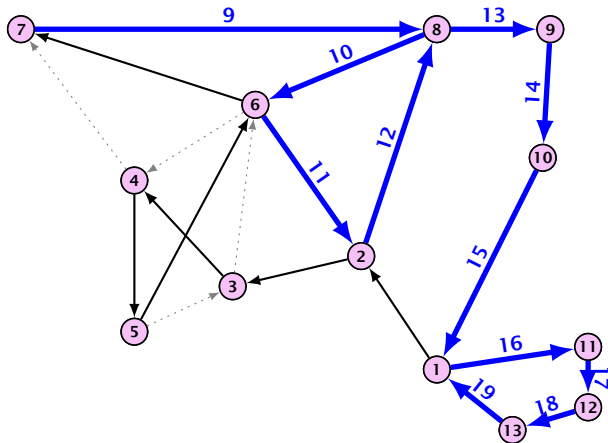
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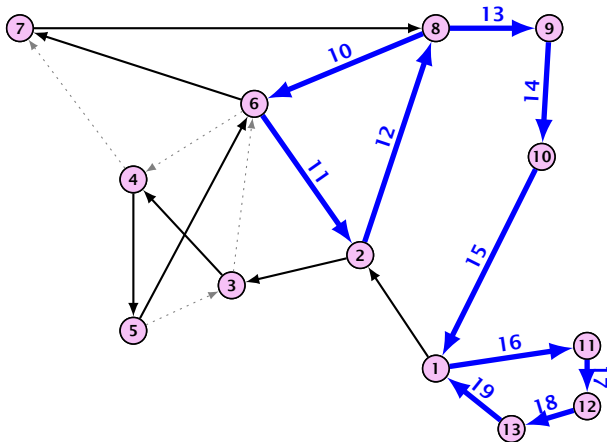
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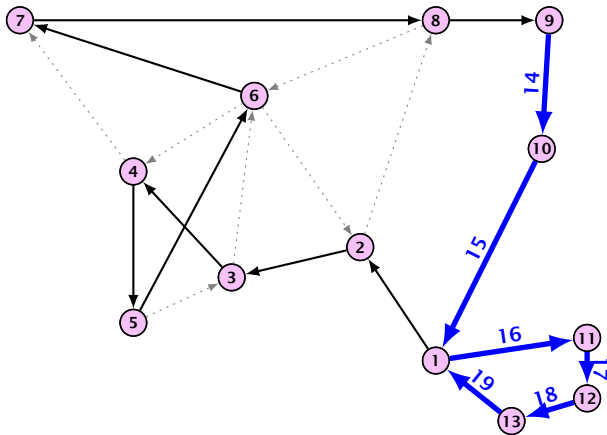
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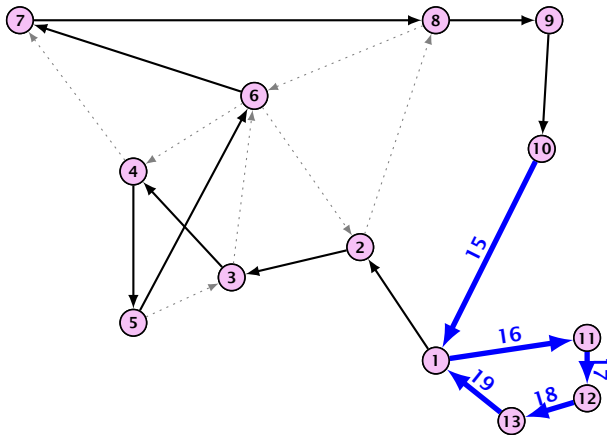
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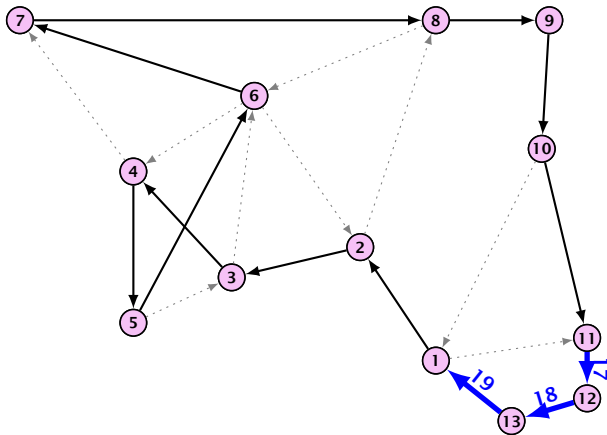
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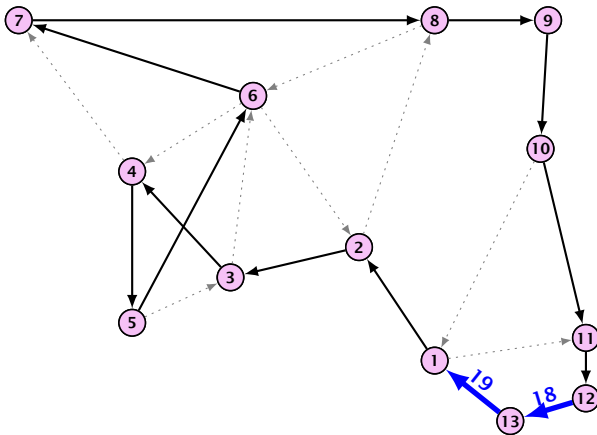
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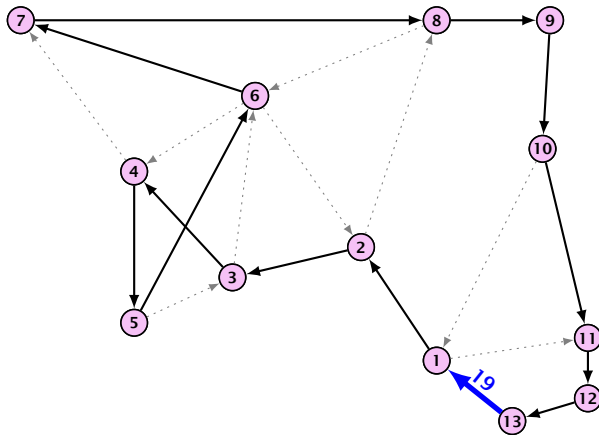
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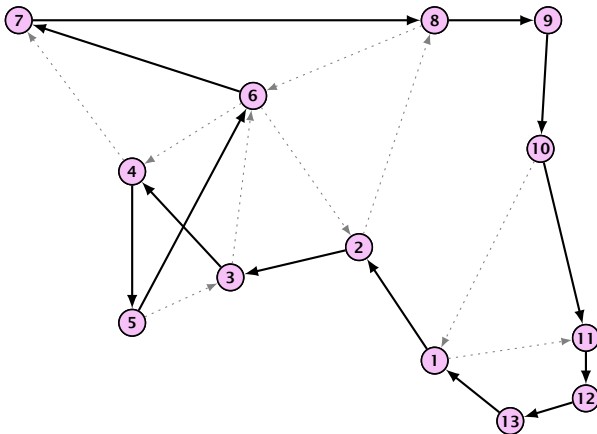
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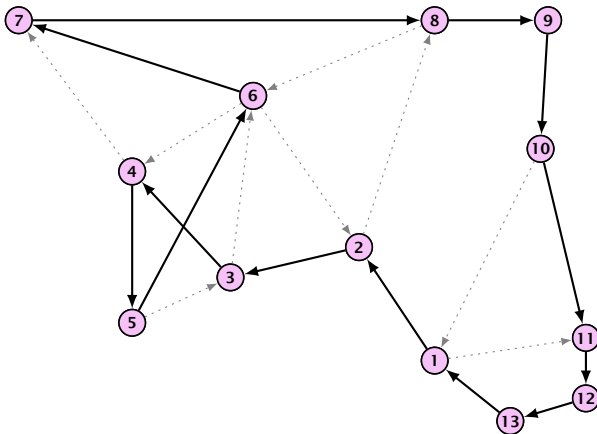
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Consider the following graph:

- ▶ Compute an MST of G .
- ▶ Duplicate all edges.

This graph is Eulerian, and the total cost of all edges is at most $2 \cdot \text{OPT}_{\text{MST}}(G)$.

Hence, short-cutting gives a tour of cost no more than $2 \cdot \text{OPT}_{\text{MST}}(G)$ which means we have a 2-approximation.

TSP: A different approach

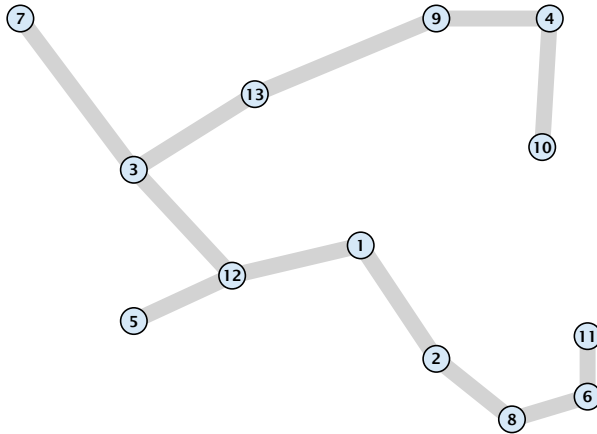
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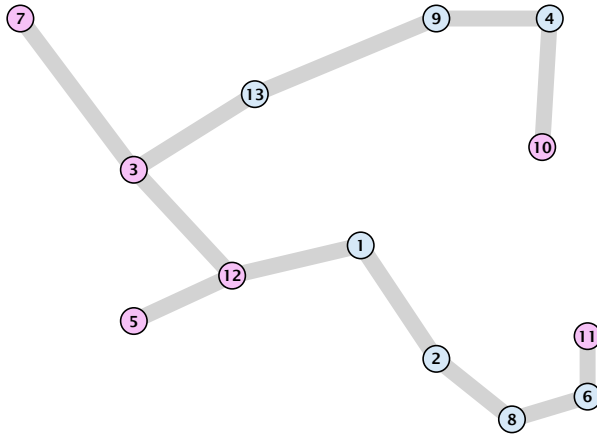
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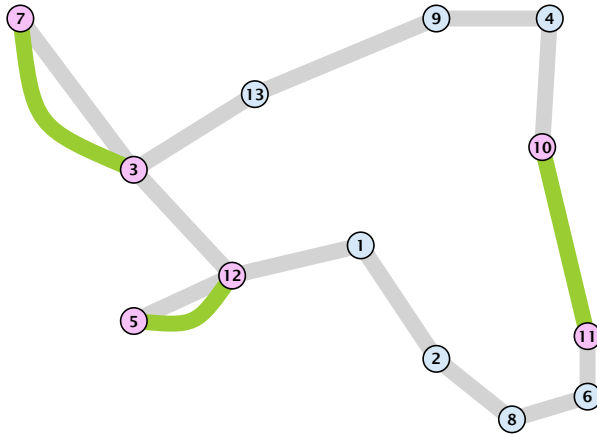
TSP: Can we do better?



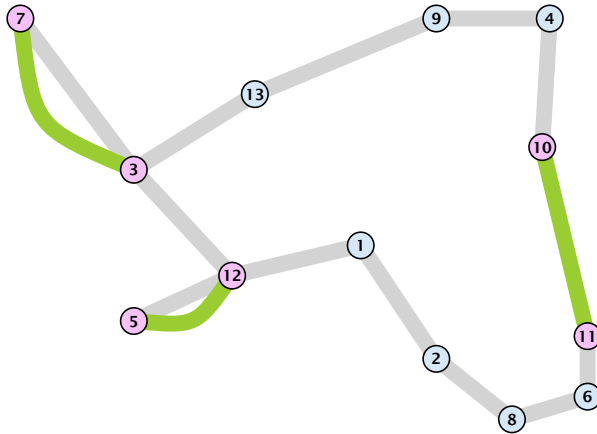
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For this we compute a Minimum Weight Matching between odd degree vertices in the MST (note that there are an even number of them).

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An optimal tour on the odd-degree vertices has cost at most $\text{OPT}_{\text{TSP}}(G)$.

However, the edges of this tour give rise to two disjoint matchings. One of these matchings must have weight less than $\text{OPT}_{\text{TSP}}(G)/2$.

Adding this matching to the MST gives an Eulerian graph with edge weight at most

$$\text{OPT}_{\text{MST}}(G) + \text{OPT}_{\text{TSP}}(G)/2 \leq \frac{3}{2} \text{OPT}_{\text{TSP}}(G) ,$$

Short cutting gives a $\frac{3}{2}$ -approximation for metric TSP.

This is the best that is known.

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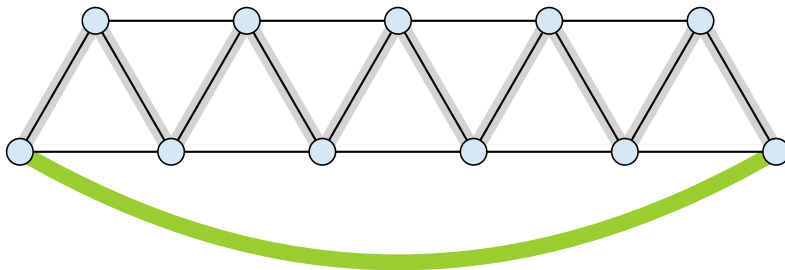
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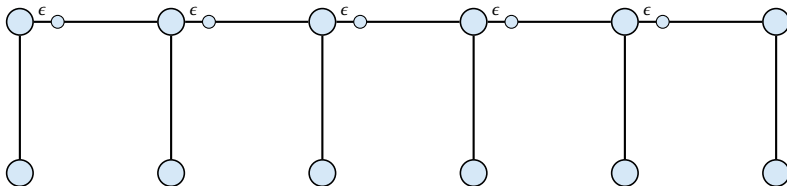
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Christofides. Tight Example



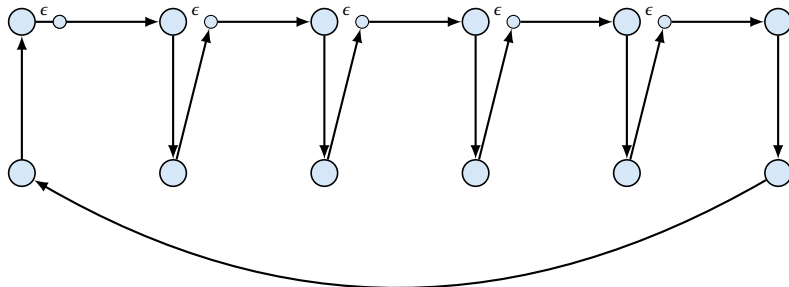
- ▶ optimal tour: n edges.
- ▶ MST: $n - 1$ edges.
- ▶ weight of matching $(n + 1)/2 - 1$
- ▶ MST+matching $\approx 3/2 \cdot n$

Tree shortcutting. Tight Example



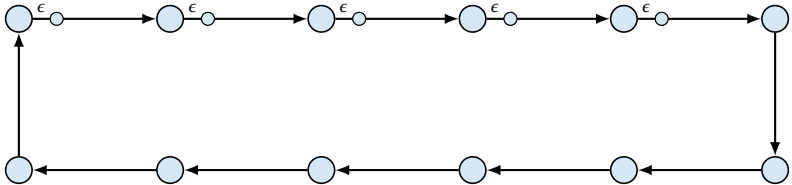
- edges have Euclidean distance.

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